Making Thinking Visible: Supporting Students in Mathematical Reasoning

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St Andrew's Junior School

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Overview

- Focus of study
- Theoretical underpinnings
- Method
- Screencast examples
- Students' perspectives
- Teachers' perspectives
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- Pedagogical implications

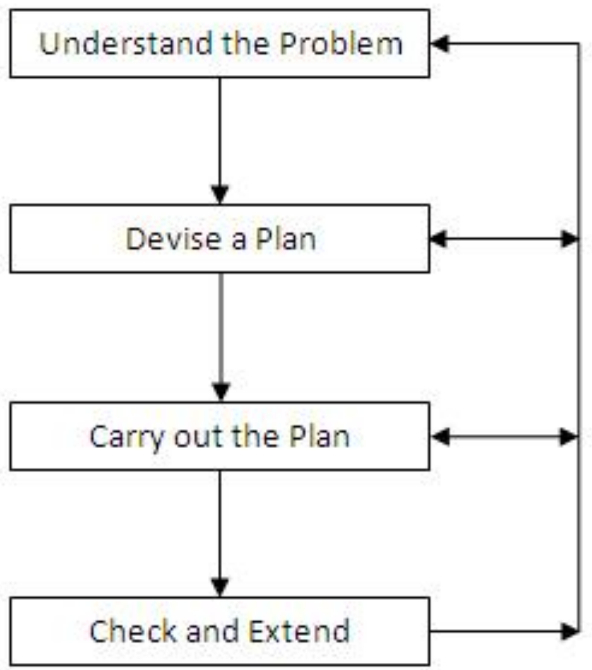
Focus of study

How teachers can support students' mathematical reasoning and make their thinking visible through oral and written communication

Specifically, the focus is on developing students' clarity in thinking by creating opportunities for strengthening students' analytical skills and verbalising their mathematical reasoning through screen casting applications.

Theoretical underpinnings

- Study adopts social constructivist approach (Perkins, 1992, Vygotsky, 1978): Learning is considered to be an active process with the learners constructing their knowledge on their own based on experience and reflecting on this experience



- Polya's problem solving model (1957)

Figure 1. Flowchart of Polya's problem-solving model

Theoretical underpinnings

- Activity-based learning (Horsburgh, 1944): learners are actively engaged in the learning process, involves reading, writing, discussion, practical activities, engagement in solving problems, analysis, synthesis, and evaluation

Verbalising thought processes: 'Think alouds'

- Explaining and teaching the steps of problem solving through self-reflection and review; demonstrating how to say aloud the steps used in problem solving (Thurlow, Barrera & Liu, 2009)

Value of Think alouds

- Effective for assessing higher-level thinking processes (those which involve working memory)
- Study individual differences in performing the same task (Olson et al.,1984)
- Encourages self-explaining - the process of developing explanations to oneself (Chi, 2000) and paraphrasing - the times the explainer repeats the text or states it in his own words
- Facilitates monitoring/metacognitive statements - Verbalizations that express the explainer's level of comprehension or what the explainer is doing or going to do (Chi et al., 1989)

Theoretical underpinnings

Screencasting technology

- allows students to create videos which serve as an effective medium for communicating mathematical understanding (Baxter, Woodward, & Olson, 2005; Casler-Failing, 2013).
- affords opportunities for authentic assessment, reflection, and higher order thinking through a rich, thoughtfully designed, and engaging activity (Richards, 2012).
- allow students to generate explanations for the purposes of learning and assessment (Soto, 2014).
- leads to richer formative assessments and the potential for improved student achievement (Walls, 2015).

Mathematics in the Singapore context

Mathematical reasoning: Ability to analyze mathematical situations and construct logical arguments.

Communication: Ability to use mathematical language to express mathematical ideas and arguments precisely, concisely and logically. Helps students develop understanding of mathematics and sharpen their mathematical thinking. (MOE, 2013, p.15)

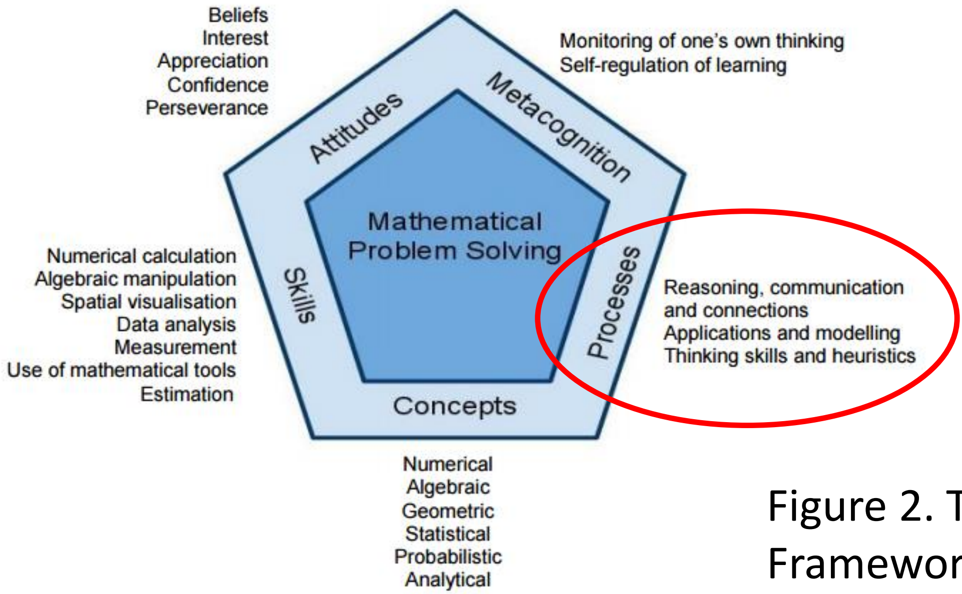


Figure 2. The Mathematics Framework (2012).

Mathematical explanations

- Mathematical explanations not only contain details how one solved the problem but also contain justifications as to why one solved the problem the way one did (Soto, 2014,p.16).

- Mathematical explanations can be communicated verbally and through a variety of mathematical symbols and representations to express mathematical understanding (Kazemi & Stipek, 2001; Schleppegrell, 2010)

Method

- Subjects
  - Mathematics Upper Primary students in one P4 class and two P3 classes (High and Middle Progress): 40 each
  - 3 teachers with 8 to 14 years of Mathematics teaching experience
  - Mainstream school, average to high socioeconomic background
- Data sources
  - Teachers’ instructional materials
  - Students’ written work
  - Screen cast recordings of lessons
  - Teachers’ feedback from individual reflections
  - Students’ feedback from surveys and focus group discussions

Method

Curriculum context:
2 cycles of Mathematics lessons, applying input from ETD and ELIS.

Each cycle consists of:

- Pre-lesson conference (Planned and framed the learning with inputs from ETD and ELIS)
- Lesson observation (Made use of the ICT-enriched pedagogical innovation & ETD officer sat in to observe, recording sent to ELIS staff for input)
- Post-lesson observation conference (Reflected upon input on teacher's instructional materials, students' screencast recordings and teacher's and students' feedback provided by ELIS & ETD)

SAJS problem solving approach (with input from ELIS)

Polya's	For Students	For Teachers
Read and Understand	Have I used Structured Questioning? Have I used chunking to identify key information? Can I restate the problem by drawing a picture or diagram to help me understand the problem?	Who (What) is/are in the story? What do they have? How many are there? Who (What) has/is more/less? What makes you say that? What happened? What did he do? What makes me say that? Are there 1 or 2 situation(s)? Who (What) is repeated? Is there a change, what is the change? If not, what remains the same? What am I trying to find out? Have I left out any other important information?
Plan	What strategy or heuristics can I use to solve the problem? What makes you say that?	Why do you think that? What convinced you? How did you come up with that answer/solution? What's your evidence for that?

SAJS problem solving approach (with input from ELIS)

Polya's	For Students	For Teachers
Carry out the Plan	Did I label my steps? Did I use the right mathematical symbols? If I am stuck, do I have an alternative method? What makes you say that?	Is that the only way to explain it? Can you think of a counter method? But what about…? Does it always work that way? Are you sure that …? Did anyone use a different approach? Who has a similar/different idea about how this works? What might be other views/solutions?
Check	Does the answer make sense? Have I used CCC to check for reasonableness and accuracy? Have I checked for calculation errors? Have I checked for transfer errors? Have I transferred information correctly? Have I included the correct standard units?	Who has the same answer as this? Who has a different solution? Are everybody's results the same? Why/why not? Have you thought of another way this could be done? Do you think we have found the best solution?

Student’s Screencast (Weak example)

Student 1

- Only reads aloud the given Q of the task
- Does not interpret demands of task.
- Merely states what S did without specifying details and reasoning for steps taken:
- ‘Draw the model, then write sentences, write final sentence’
- Does not show explicitly understanding of mathematical operations/model involved with the lack of specific and details in what was written down
- Does not show clear link between what was drawn visually and what was verbalised without the specifics
- Does not account/explain/justify the ‘why’ for ‘what’ was done
- Imprecise use of language: ‘sentences’ used to indicate the mathematical equation or formula

Student's Screencast (Strong example)

Student 2

- Reads aloud Q as well as interprets Q demands in S's own words
- States model to be used – 'comparison model' (content vocab)
- De-constructs/breaks down explicitly the steps involved in dealing with one model after another – 'the first' followed by 'the second'- which cumulatively builds up to the final solution
- Explains visual representation by explicitly stating what S does and linking it to explaining why and how each step is necessary: 'As you can see'
- Appropriate use of specific language features:
  - (functional language) that provide coherence to explanation:
    - -'So, in order to find………
    - Now…. on the empty box… which will give us
    - Therefore, the mass of the empty box is….'
- Mathematical operation-content vocab 'minus'

Students' perspectives

How verbalizing thought processes helped students	Example of student response
Explains reasoning and decisions (Analytical)	… So this question is Lucas and Shiro had 3678 altogether. After Lucas gave $34 to Shiro, he had 5 times as much as Shiro. How much did Shiro have at first? So, we know over here that Lucas and Shiro had 3678 and Lucas gave $34 to Shiro. And then, he had… Lucas had 5 times as much as Shiro. So since there's no… there's no money taken off the people in this question, this is an internal transfer method.

Students' perspectives

How verbalizing thought processes helped students	Example of student response
Thinks of ways to improve (Reflective)	… Now we have to find, now we have to find 1 unit, 1 U equal 16 divided by 2, which is 8. Since this we have to find how many women at the session at first which is 3 unit, so 1 U times 3 equal which is 24, so 24 is our answer. There were 24 women at the Session at first. Now we have to check. 24 divided by 3 equal 8. So 8 and 8 is correct. 8 times 2 equal 16. 16 and 16 is correct. 16 minus second, something is wrong, hai, minus 12 equal 4 and correct. So everything is tick, tick, tick.

Quality of students' mathematical reasoning

Quality of mathematical reasoning	Frequency (N=41)	Percentage
Strong	27	66
Weak	14	34

Aspects of:

- Accuracy in use of mathematical language
- Ability to deconstruct demands of given tasks
- Ability to identify problem solving method to be used
- Clarity in articulating mathematical reasoning

Teachers’ perspectives

Aspect	Teacher response
Teachers’ beliefs in value of screencast technology	Screencasts offered valuable opportunities for students to construct solutions for themselves through classroom talk and the use of think-aloud strategy. Use of ICT (screen casts) in capturing students’ talk allows maximum participation and increased engagement for everyone in the classroom, even for shy students.

Teachers' perspectives

Aspect	Teacher response
Teachers' beliefs in value of classroom talk and effective questioning	Classroom talk can be used to guide the development of students' mathematical argumentation and problem solving skills. Classroom talk and focus on oral language can promote students' intellectual development. Effective questioning and classroom talk helps students see the connections between mathematical concepts without telling them too much.

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Teachers’ perspectives

Impact on teachers’ practice	Teacher response
Awareness of structuring talk in order to making students’ thinking visible	Questions need to be structured so as to provoke thoughtful answers. Students’ responses can stimulate further questions and are seen as building blocks to learning rather than as terminal points. Teacher-student and student-student talk can be chained into coherent lines of enquiry rather than remain disconnected.

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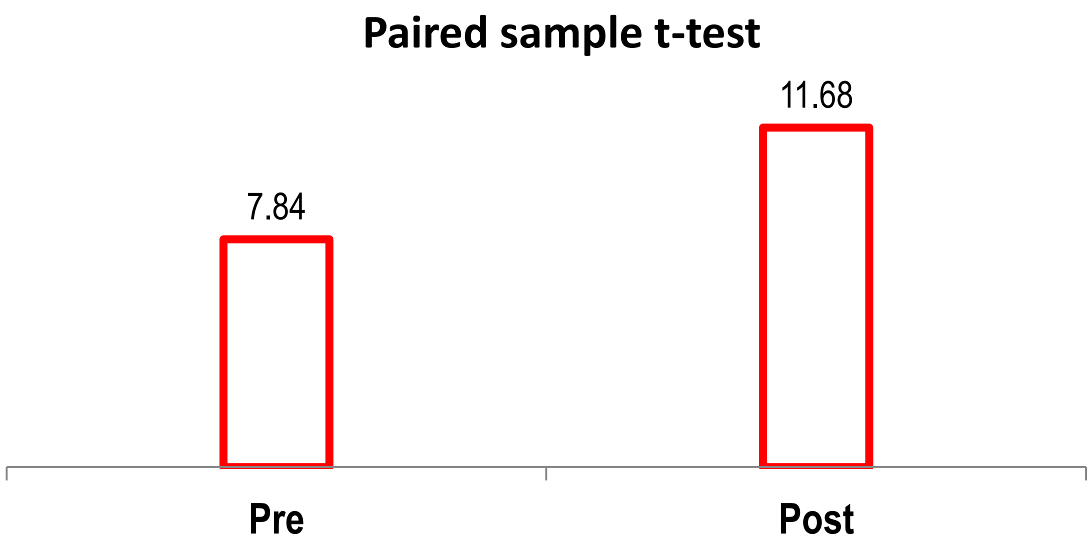
Teachers' perspectives

Impact on teachers' practice	Teacher response
Awareness of specific aspects of practice that need to be emphasised	Teachers role modelling the use of language as a tool of thinking is important. Extra steps needed to create opportunities for students' talk in the classroom. Carefully planned activities and thought-provoking questions in a supportive environment facilitate mathematical reasoning and sense-making, mathematical problem solving.

Quantitative results



Class	1
N	37
t-value	5.935
Significance value	.000
Significant Improvement	✓

Paired Sample t-test of pre- and post-test scores

Pedagogical implications

- Deliberate planning of time and space for students to articulate their views
- Designing of learning activities that encourage the use of talk to support learning
- Incorporate the use of screencasting applications
  - Allows students to make their mathematical thinking and reasoning visible
  - Screencasts facilitate peer assessment for collaborative learning
- Sustained practice to reinforce critical skills, and internalise and process learning

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